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Short Note

Applying a Change-Point Detection Method on Frequency-Magnitude Distributions

by Daniel Amorèse

Abstract A method based on nonparametric statistics (hereafter called "median-based analysis of the segment slope," MBASS) is applied for the detection of change points in frequency magnitude distributions (FMDs). The determination of the lowest magnitude for which the Gutenberg–Richter relation still applies is a key point for the computation of reliable *b*-values. The change-point detection method presented here is used to determine automatically this threshold magnitude (called m_0 in this study) for large subsets extracted from four seismic catalogs. Results are successfully compared to those of a previous benchmark study from other authors. Moreover, the MBASS procedure is able to detect a magnitude artifact in the FMDs of a regional catalog. The results of the MBASS procedure confirm that a break in slope in a cumulative frequency distribution may be misleading when FMDs are analyzed by eye.

Ⓔ *Online material:* Source code for detection of change-points and NIED sample data set.

Introduction

Earthquake frequency-magnitude distribution (FMD) shows the Gutenberg and Richter relation (Gutenberg and Richter, 1944) between the frequencies and magnitudes of earthquakes. This fundamental statistical description of seismicity is expressed by:

$$\log_{10} N(M) = a - bM \qquad (1)$$

where a and b are constants, M is the magnitude, and $N(M)$ is the number of earthquakes that occur in a specific time window with magnitude $\geq M$. This is the cumulative version and the more widespread expression of the Gutenberg–Richter law. The incremental distribution where N is the number of earthquakes with magnitudes in a fixed range around M is poorly used for the computation of *b*-values. The *b*-value in the Gutenberg–Richter power law is an essential tool in seismotectonic studies and seismic-risk analysis. Therefore, its correct computation represents an important challenge for seismology.

Many difficulties arise in calculating the *b*-value (Chan and Chandler, 2001; Felzer, 2006), among which the main one is certainly the existence of breakpoints in the FMD (Fig. 1). The most obvious breakpoint is induced by a deviation from the power law at the low-magnitude end. For some authors (Wiemer, 2001; Woessner and Wiemer, 2005), this point corresponds to the magnitude of completeness M_c, which is defined as the lowest magnitude at which 100% of the events in a space–time volume are detected (Rydelek and Sacks, 1989). Nevertheless, many other interpretations of the discrepancies at the low-magnitude end imply a departure from self-similarity for the smallest earthquakes (Aki, 1987; Main, 1987; Taylor *et al.*, 1990; Speidel and Mattson, 1993). Thus, although this point is still disputed (Rydelek and Sacks, 2003), in this study, in common with earlier authors when computing the *b*-value (Aki, 1965; Utsu, 1965; Shi and Bolt, 1982; Tinti and Mulargia, 1987), the low-magnitude breakpoint of the FMD is not called M_c but m_0. This threshold magnitude is required for reliable computation of the *b*-value. Many authors have addressed the estimation of this magnitude (Rydelek and Sacks, 1989; Ogata and Katsura, 1993; Wiemer and Wyss, 2000; Cao and Gao, 2002; Marsan, 2003; Woessner and Wiemer, 2005).

At the other end of the FMD, toward the largest magnitudes, deviations from the simple log linear Gutenberg–Richter law are often expected (Main and Burton, 1986; Pacheco *et al.*, 1992; Scholz, 1997; Aki, 2000; Stock and Smith, 2000; Lasocki and Papadimitriou, 2006). However, this point also is not settled and challenging explanations have been proposed (Howell, 1985; Sornette *et al.*, 1996; Main, 2000). Except for Lasocki and Papadimitriou (2006),

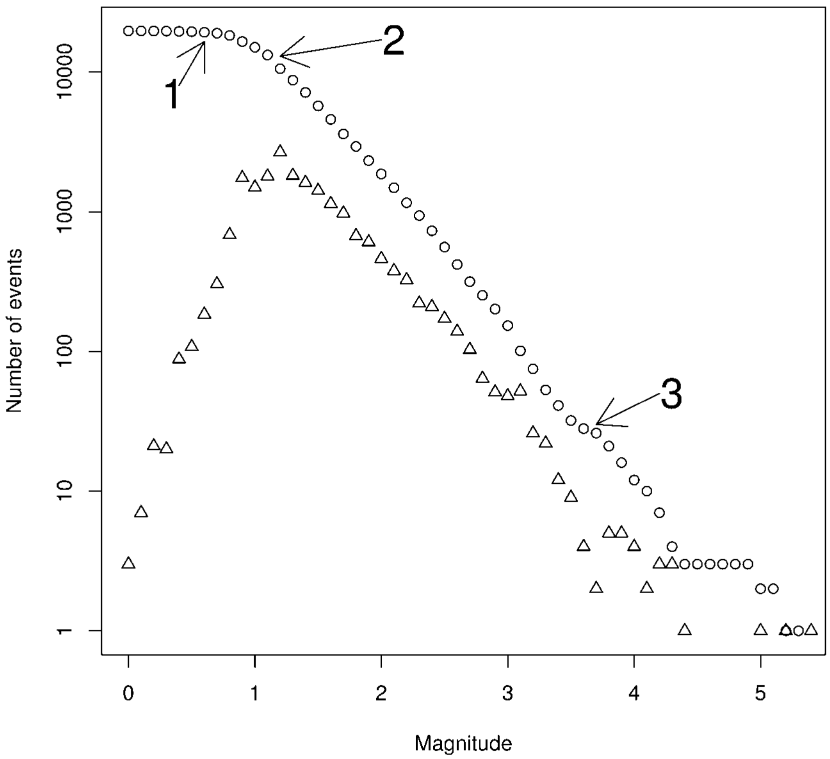

Figure 1. Examples of frequency magnitude distributions from a studied data set (NCSN). The numbered arrows mark typical apparent breakpoints of the cumulative distribution. Typically, breakpoint 2 has m_0 magnitude.

who used a nonparametric approach, investigators have so far addressed this question by using goodness-of-fit tests. These procedures do not provide strong indications on the high significance of the null statistical hypotheses (Lasocki and Papadimitriou, 2006).

In this study, a quick and new procedure is presented that uses nonparametric statistics to allow the analysis of FMDs on their entire magnitude range. Thus, this procedure addresses both the determination of m_0 and the detection of a possible upper-magnitude breakpoint. Nevertheless, questions dealing with the origins and implications of the possible upper-magnitude breakpoint are not addressed in detail in this short methodological article. Instead, I limit the application of my procedure to well-studied FMDs. I show that the automatic detection of significant deviations in FMDs is possible. About the determination of uncertainties in the breakpoint locations, I follow Woessner and Wiemer (2005) and choose a bootstrap approach. This is also a nonparametric technique where the confidence intervals are explicitly estimated.

This study shows that my procedure

1. is relevant for the m_0 determination.
2. is not duped by apparent breakpoints in cumulative FMDs.
3. can be used to highlight magnitude artifacts in a data set.

Data

My study considers the work of Woessner and Wiemer (2005) as a benchmark study. In their article, Woessner and Wiemer (2005) (W&W, 2005) estimated M_c values (in this study called m_0) by different procedures for several data sets. W&W (2005) show a summary of their results (M_c and b-value determinations) for four freely downloadable data sets (W&W, 2005; Table 1). In this study, I used the same four data sets. These data sets were obtained from:

1. a regional catalog: a subset of the Earthquake Catalogue of Switzerland (ECOS) from the Swiss Seismological Service (SSS).
2. a regional catalog: a subset of the Northern California Seismic Network (NCSN), U.S. Geological Survey, Menlo Park (investigated magnitudes are NCSN coda-duration magnitude values).
3. a volcanic region: a subset of the earthquake catalog maintained by the National Research Institute for Earth Science and Disaster Prevention (NIED).
4. a global dataset: a subset of the Global Centroid Moment Tensor (CMT) catalog (investigated magnitudes are M_w values that have been converted from seismic moments).

Possibly due to (1) incomplete descriptions in the text of their article and/or (2) to changes in the catalogs, it appears that none of the four W&W datasets (2005) is exactly reproducible from the presently downloadable raw catalogs. For instance, concerning the NIED data set, in the W&W (2005) version, all the events with magnitudes smaller than zero are missing. The differences in the number of events can be estimated with Table 1. To promote reproducibility, all the data sets used in this study are exactly described in Table 1.

Table 1
Parameters of the Data Sets, M_c, M_0, and b-values

	Catalog			
	ECOS	NIED	CMT	NCSN
Number of events	985	31,240	16,472	19,690
Number of events (W&W, 2005)	988	30,882	16,385	19,559
Minimum longitude	6.81° E	138.9514° E	179.98° W	122.9992° W
Maximum longitude	8.39° E	139.3499° E	180.00° E	120.5020° W
Minimum latitude	45.91° N	34.8002° N	72.41° S	36.0002° N
Maximum latitude	46.64° N	35.0499° N	87.02° N	38.9993° N
Maximum depth	19 km	205.3 km	689 km	75.1 km
Start date	01/13/1992	01/01/1992	01/01/1983	01/01/1998
End date	12/28/2002	12/28/2000	12/31/2002	12/31/2000
Minimum magnitude	0.5	−0.4	M_w 4.8	M_d 0.0
Maximum magnitude	4.4	5.1	M_w 8.4	M_d 5.4
M_c (W&W, 2005)	1.5 ± 0.13	1.25 ± 0.05	5.39 ± 0.04	1.20 ± 0.07
M_0 (mean, 90% CI)	1.43 ± 0.25	1.32 ± 0.16	5.42 ± 0.08	1.20 ± 0.02
M_0 (percentiles)	1.4 (1.3–1.7)	1.3 (1.2–1.5)	5.4 (5.4–5.5)	1.2 (1.2–1.2)
b (W&W, 2005)	0.96 ± 0.07	0.81 ± 0.02	0.89 ± 0.01	0.98 ± 0.02
b (percentiles)	0.91 (0.84–1.05)	0.83 (0.80–0.87)	0.99 (0.97–1.00)	0.94 (0.93–0.96)

Percentiles are respectively, the 50th (median), 5th, and 95th percentiles of the bootstrap empirical distributions. The 90% confidence intervals are calculated by multiplying each standard deviation value by 1.645.

In their study, for each data set, Woessner and Wiemer (2005) computed both M_c and b-values. The clearest way to test my procedure is to try to reproduce these results. Anyway, using synthetically created data sets is not really relevant because doubts exist on the theoretical shape of FMDs.

Many equations have been developed and discussed to estimate b either from continuous or incremental data (Aki, 1965; Utsu, 1965; Page, 1968; Weichert, 1980; Bender, 1983; Tinti and Mulargia, 1985). Following Woessner and Wiemer (2005), I use the Aki–Utsu equation with continuity correction (in this study, all the magnitude values have been rounded to one decimal place):

$$b = \log_{10}(e)/[\bar{m} - (m_0 - 0.05)]. \tag{2}$$

Testing the relevancy of my procedure for the detection of the upper-magnitude breakpoint is not trivial because the phenomenon is not always expected and benchmark studies are missing. Indeed, as its existence is discussed, this special point of the FMD is, in general, never automatically computed by FMD investigators. In this study, I assume that if my procedure works well for m_0, there is no reason for it to fail for another discontinuity. Readers are set free to make their own opinions about the median-based analysis of the segment slope (MBASS) procedure with their own data sets, using the source code in the Appendix.

Method

To detect the m_0 point, the multiple change-point procedure (Lanzante, 1996) was adopted. This procedure, fully described in Lanzante (1996), is an iterative method designed to search for multiple change points in an arbitrary time series. This method, which uses resistant, robust, and nonparametric statistical techniques, has already been applied successfully for the analysis of climate data (Lanzante, 1996; Lanzante *et al.*, 2003). Lanzante's method is based on the change-point test (Siegel and Castellan, 1988) to determine, and locate when, a point in the time series at which the median changes. In this study, Lanzante's method was applied on segment slopes of the incremental FMD. If M_1 and M_2 are the magnitudes of two consecutive points of the FMD, respectively, the segment slope for $M = M_2$ was defined as:

$$s(M_2) = \frac{\log[N(M_1)] - \log[N(M_2)]}{M_1 - M_2} \tag{3}$$

The segment slope was computed for each magnitude increment. For this reason, the method here is termed MBASS. Moreover, in MBASS, the time line is replaced by the "magnitude line": for each FMD, I looked for any magnitude value corresponding to a significant and stable change in the median of the segment slope of the FMD.

At each point i in the magnitude series of n points (segment slope), the sum of the ranks from the beginning of the series to that point is computed. This sum (SR_i) is adjusted (SA_i):

$$SA_i = |(2SR_i) - i(n + 1)|. \tag{4}$$

The next step was to find the maximum of the adjusted sum. If the maximum value of the SA_i is located at point n_1, the following variables are defined:

$$W = SR_{n1} \tag{5}$$

$$n_2 = n - n_1. \tag{6}$$

The acceptance of rejection of the null hypothesis that there is no change in the sequence at n_1 is based on the widely used Wilcoxon–Mann–Whitney (WMW) nonparametric test. This test was also referred to as Wilcoxon rank sum test (Wilcoxon, 1945) or the Mann–Whitney *U* test (Mann and Whitney, 1947). The MBASS procedure cannot be applied to cumulative FMD data because, in common with many other statistical procedures, the WMW test requires independence within groups (Hollander and Wolfe, 1973). The change-point test was applied to the magnitude series as long as the statistical significance of each new change point was less than a specified level (for example, in this study, the chosen level of significance was 5%). For each iteration, a list of *N* change points was produced that defined $N + 1$ change-point segments. At each iteration, the magnitude series was adjusted by subtracting the median of its segment from each point. The next change-point test was applied to the adjusted magnitude series.

For each FMD, Lanzante's method may find several significant change points. In this study, only two discontinuities were investigated:

1. The main discontinuity (Figs. 2B, 3B, 4B, and 5B) is the change point that corresponds to the smallest probability of making an error when rejecting the null hypothesis (type 1 error).
2. The "auxiliary" discontinuity (Figs. 2C, 3C, 4C, and 5C) is the breakpoint that corresponds to the first relative minimum value of the type 1 error probability.

To be consistent in the nonparametric approach, I used nonparametric bootstrap (Efron and Tibshirani, 1993) percentile confidence intervals to infer the uncertainty in the estimate of magnitude breakpoints. The bootstrap distribution of magnitude discontinuities was obtained from 1000 bootstrap replicates and the 5th and 95th percentiles formed the limits for the 90% bootstrap confidence interval. To make comparison easier with Woessner and Wiemer's study (2005), 90% confidence interval for the mean M_0 were also computed from standard deviations (Table 1).

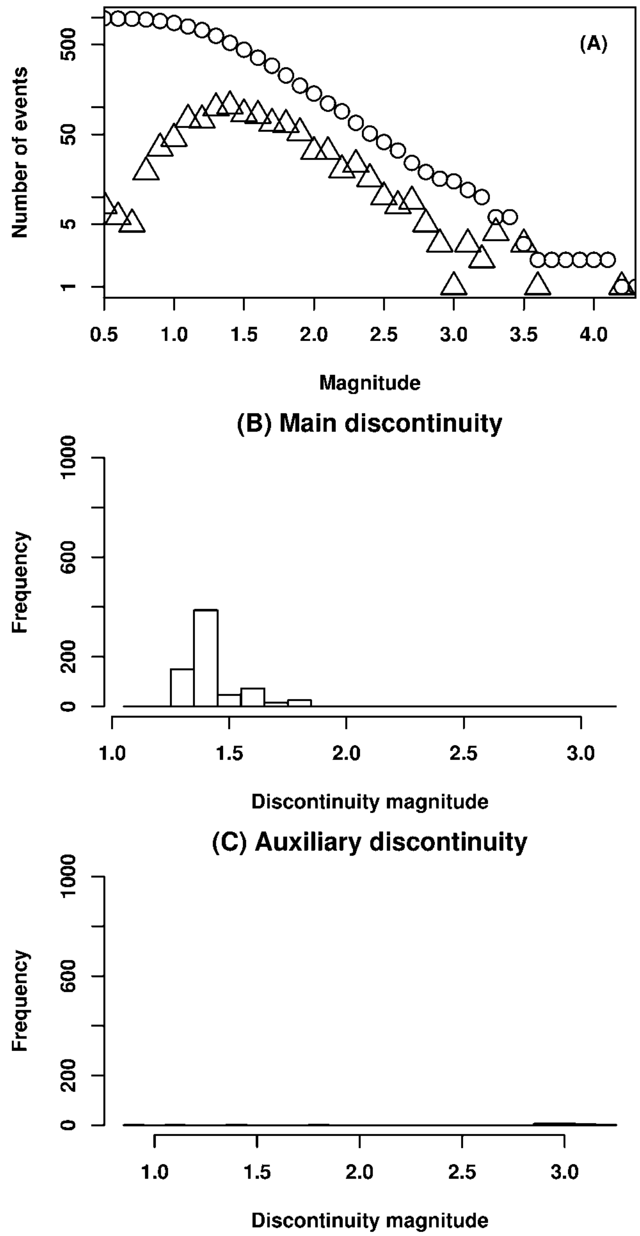

Figure 2. Frequency magnitude distributions and histograms of the magnitude discontinuities for the SSS data set. (a) Cumulative (circles) and incremental (triangles) FMDs. (b) Histogram showing the distribution of the main MBASS magnitude discontinuity (m_0) obtained from 1000 bootstrap replicates. (c) Same as (b) but for the auxiliary discontinuity.

Results

Results from the MBASS method are compared with those of Woessner and Wiemer's study (W&W, 2005) (Table 1). The value of m_0 corresponds to the main discontinuity magnitude. When uncertainties are considered, most of the results are matching with each other (Table 1). The only important discrepancy is observed for the *b*-values computed from the CMT data sets. For the CMT catalog, I believe that a *b*-value close to 1 is the correct value. Note that a recent study provides the same result for *b* (Felzer, 2006).

Therefore, the MBASS procedure is relevant for the determination of the FMD threshold magnitude to calculate correct *b*-values. The other use of MBASS is the detection of additional discontinuities in FMDs. Even if their cumulative FMDs show apparent #3 type breakpoints (breakpoint types are shown in Fig. 1), the SSS, NIED, and CMT data sets did not show significant discontinuities when their incremental distributions were investigated by the MBASS procedure (Figs. 2, 3, and 4). This observation agrees with the statement of Main (Main, 2000): "it is unwise to interpret the cumulative frequency data uniquely in terms of a break in slope, if there is no apparent break of slope in the incremental distribution." For the NCSN data set, MBASS detected a

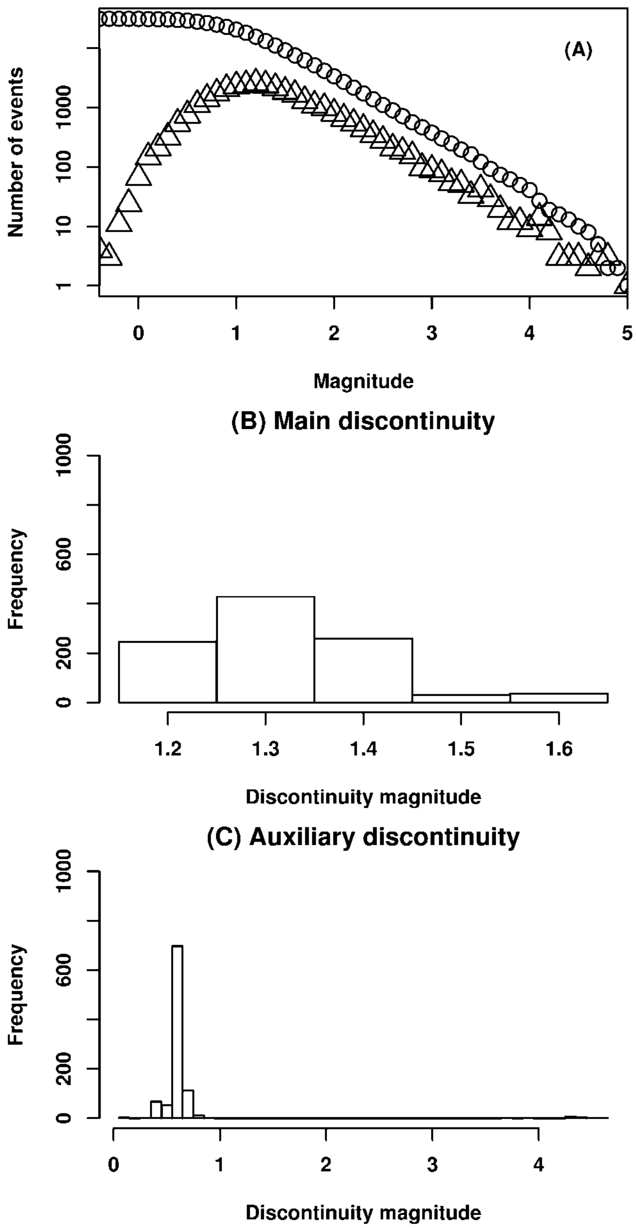

Figure 3. Frequency magnitude distributions and histograms of the magnitude discontinuities for the NIED data set. See legend to Figure 2.

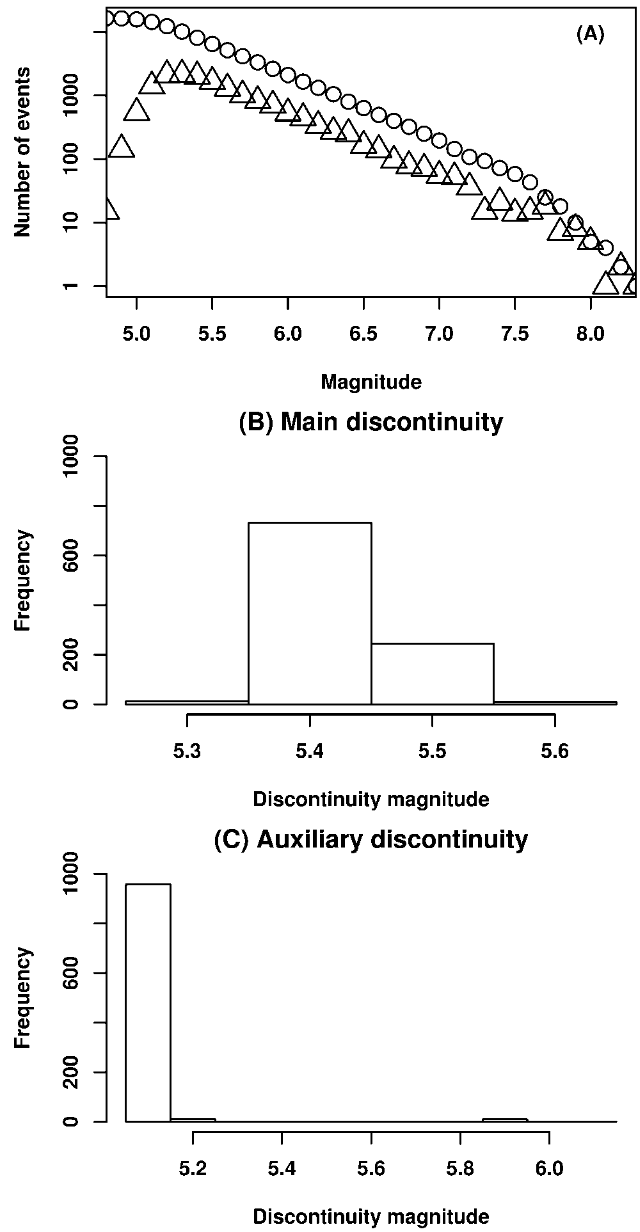

Figure 4. Frequency magnitude distributions and histograms of the magnitude discontinuities for the CMT data set. See legend to Figure 2.

significant (about 150 replicates) auxiliary discontinuity near M 3.8 (Fig. 5). The reason for this anomaly is possibly that the number of events being calculated as M_d is changing around this magnitude (K. R. Felzer, pers. comm., 2006). Thus, the MBASS procedure can also be useful for detecting discontinuity artifacts in FMDs.

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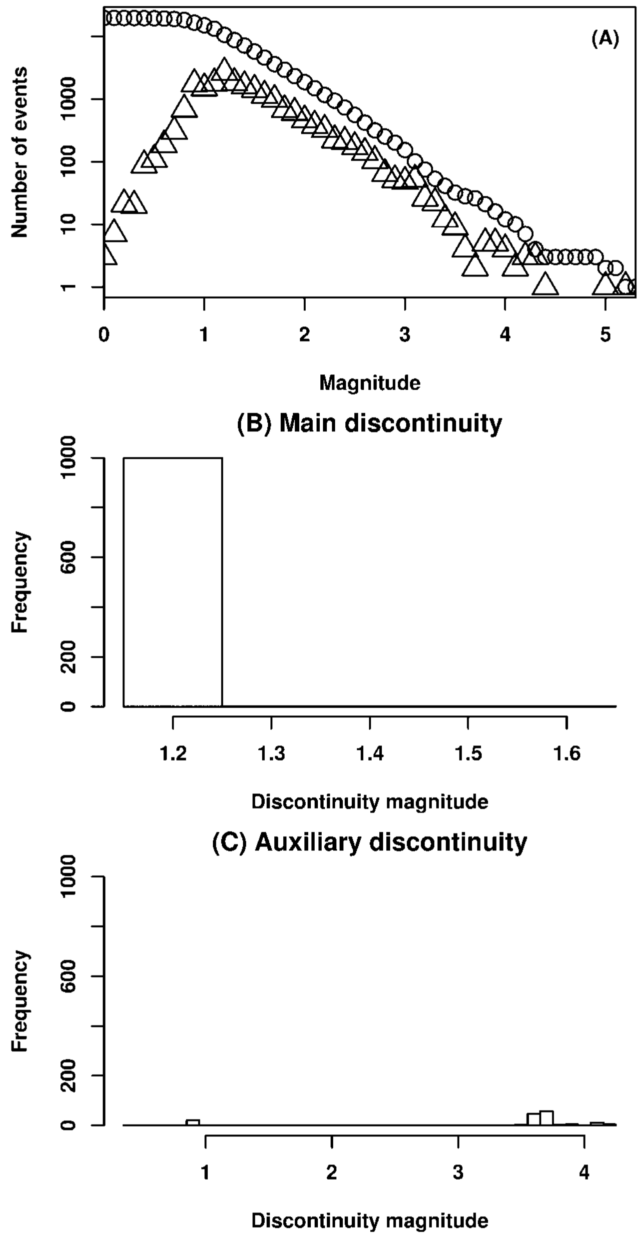

Figure 5. Frequency magnitude distributions and histograms of the magnitude discontinuities for the NCSN data set. See legend to Figure 2.

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Appendix A

This appendix gives the R source code for the detection of change points. (Ⓔ The R source code is available in the electronic edition of BSSA.) It needs a list of magnitude values in vector “a.” The “fmbass” function performs the raw MBASS procedure. The “mbass” function performs the bootstrap of “fmbass.” The version of the source code is valid for R ≥ 2.4.0. R is the free statistical programming language (Ihaka and Gentleman, 1996) that serves to write the Statistical Seismology Library (Harte, 2006). In this listing, the bootstrap library is the 1.0-20 bootstrap package, which includes the functions for the book by Efron and Tibshirani (1993).

```
library (bootstrap)
"fmbass" <-
function (a, delta=0.1, plot=TRUE, alldisc=
  FALSE)
{
if (plot) {par (mfrow=c(1, 1))}
tau <- numeric ()
pva <- numeric ()
minmag <- min (a, na.rm=T)
g_r <- hist (a, plot=F, breaks=seq ((minmag-
  delta/2), (max (a, na.rm=T) +delta/2),
  delta))
n <- length (g_r$intensities)
xc <- seq (minmag, max (a, na.rm=T), delta) [1:
  (n-1)]
log_nc <- log10 ((1/delta) * (length(a) - cumsum
  (g_r$counts) [1: (n-1)]) *delta)
x <- seq (minmag, max (a, na.rm=T), delta)
log_n <- log10 ((1/delta) *g_r$counts*delta)
x <- x [is.finite (log_n)]
log_n <- log_n [is, finite (log_n)]
sl <- diff(log_n)/diff(x) #segment slopes
xsl <- x[2:length(x)]
if (plot) {
plot(xc, (10^log_nc), type="p", ylim=c (1,
  length (a)), log="y", xlab="Magnitude",
  ylab="Number of events", pch=1)
points (x, (10^log_n), pch=2)
}
niter <- 3
N <- length(sl)
j <- 0 #iterations
k <- 0 #discontinuities
SA <- vector(length=N)
while (j < niter) {
 for (i in seq (1,N,1)) SA[i] <- abs (2*sum (rank
   (sl) [1:i]) -i* (N+1))
 n1 <- which (SA==SA[order(SA) [length (order
   (SA))]])
 xn1 <- sl [1:n1 [1]]
 xn2 <- sl [- (1:n1 [1])]
 if ( (n1 [1]>2) && (n1 [1] <= (N-2) ) && (wilcox.
   test (xn1, xn2, exact=F, correct=T) [3]
   <0.05) ) {
  k <- k+1
  pva [k] <- wilcox.test (xn1, xn2, exact=F,
   correct=T) [3]
  tau[k] <- n1 [1]
  if (k>1) {
    medsl1 <- median (sl [1:n0])
    medsl2 <- median (sl [- (1:n0)])
    for (i in seq (1, n0, 1)) sl [i] <- sl
        [i]+medsl1
    for (i in seq(n0+1, length(sl), 1)) sl [i]
        <- sl [i]+medsl2
}
  medsl1 <- median(sl[1:n1[1]])
  medsl2 <- median(sl[-(1:n1[1])])
  for (i in seq(1, n1[1], 1)) sl[i] <- sl [i]-
    medsl1
  for (i in seq(n1[1]+1, length(sl), 1)) sl[i]
    <- sl[i]-medsl2
  n0 <- n1[1]
}
 j <- j+1
}
v_pva=as. vector (pva, mode="numeric")
ip=order (v_pva)
m0=c(signif(xsl[tau[ip[1]]]),
  signif(xsl[tau[ip[2]]]))
```

```
if(alldisc) {return (print (list (discmag=xsl
  [tau], p=v_pva, m0=m0)))}
invisible (m0)
}
"mbass" <-
function (a, delta=0.1, plot=TRUE, alldisc=
  FALSE, bs=0)
{
mba <- function (x) {fmbass (x, delta, plot,
  alldisc)}
if (bs==0) {res <- mba (a)} # actual FMD
  analyzed
else {
res=bootstrap (a, abs (bs), mba) # bs is the
  number of bootstrap replicates
}
invisible (res)
}
```

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